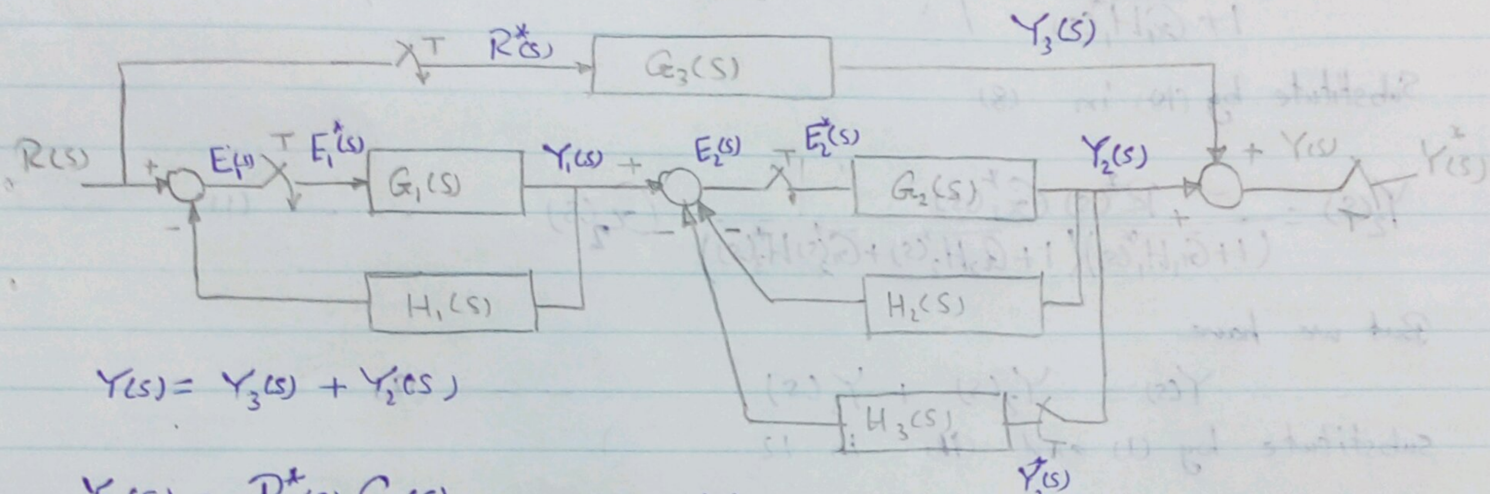
Digital Control Midterm Exam

Model Answer

Problem no.(1) part (a)



$$Y(s) = Y_3(s) + Y_2(s)$$

$$Y_3(s) = R^*(s)\, G_3(s) \quad \text{----} \quad (1)$$

$$E_1(s) = R(s) - H_1 Y_1(s) \quad \text{----} \quad (2)$$

$$Y_1(s) = E_1^*(s)\, G_1(s) \quad \text{----} \quad (3)$$

Substitute by (3) in (2)

$$E_1(s) = R(s) - G_1H_1(s)\, E_1^*(s) \quad \text{----} \quad (4)$$

staring of (4)

$$E_1^*(s) = R^*(s) - \overline{G_1H_1}^*(s)\, E_1^*(s)$$

$$\therefore E_1^*(s) = \frac{R^*(s)}{1+\overline{G_1H_1}^*(s)} \quad \text{----} \quad (5)$$

substitute by (5) in (3)

$$Y_1(s) = \frac{R^*(s)\, G_1(s)}{1+\overline{G_1H_1}^*(s)} \quad \text{----} \quad (6)$$

$$E_2(s) = Y_1(s) - H_2 Y_2(s) - H_3(s)\, Y_2^*(s) \quad \text{----} \quad (7)$$

$$Y_2(s) = E_2^*(s)\, G_2(s) \quad \text{----} \quad (8)$$

Substitute by (6) and (8) in 7

$$E_2(s) = \frac{R^*(s)\, G_1(s)}{1+\overline{G_1H_1}^*(s)} - G_2H_2(s)\, E_2^*(s) - G_2^*(s)\, H_3(s)\, E_2^*(s) \quad \text{----} \quad (9)$$

staring of (9)

$$E_2^*(s) = \frac{R^*(s)\,G_1^*(s)}{1+\overline{G_1H_1}^*(s)} - \overline{G_2H_2}^*(s)\,E_2^*(s) - G_2^*(s)\,H_3^*(s)\,E_2^*(s)$$

$$E_2^*(s) = \frac{R^*(s)\,G_1^*(s)}{1+\overline{G_1H_1}^*(s)} \Big/ \left(1+\overline{G_2H_2}^*(s) + G_2^*(s)\,H_3^*(s)\right) \quad \text{--- (10)}$$

Substitute by (10) in (8)

$$Y_2(s) = \frac{R^*(s)\,G_1^*(s)}{\left(1+\overline{G_1H_1}^*(s)\right)\left(1+\overline{G_2H_2}^*(s)+G_2^*(s)\,H_3^*(s)\right)}\,G_2(s) \quad \text{---- (11)}$$

But we have

$$Y(s) = Y_2(s) + Y_3(s) \quad \text{--- (12)}$$

substitute by (1) and (11) in 12

$$\therefore Y(s) = \frac{R^*(s)\,G_1^*(s)\,G_2(s)}{\left(1+\overline{G_1H_1}^*(s)\right)\left(1+\overline{G_2H_2}^*(s)+G_2^*(s)\,H_3^*(s)\right)} + R^*(s)\,G_3(s) \quad \text{--- (13)}$$

staring of (13)

$$Y^*(s) = \frac{R^*(s)\,G_1^*(s)\,G_2^*(s)}{\left(1+\overline{G_1H_1}^*(s)\right)\left(1+\overline{G_2H_2}^*(s)+G_2^*(s)\,H_3^*(s)\right)} + R^*(s)\,G_3^*(s)$$

$$\therefore \frac{Y^*(s)}{R^*(s)} = \frac{G_1^*(s)\,G_2^*(s)}{\left(1+\overline{G_1H_1}^*(s)\right)\left(1+\overline{G_2H_2}^*(s)+G_2^*(s)\,H_3^*(s)\right)} + G_3^*(s) \quad \text{--- (14)}$$

from (14) The closed loop digital transfer function is:

$$\boxed{\frac{Y(z)}{R(z)} = \frac{G_1(z)\,G_2(z)}{\left(1+G_1H_1(z)\right)\left(1+G_2H_2(z)+G_2(z)H_3(z)\right)} + G_3(z)}$$

Problem no (2) Part (b)

i) c/c eqn $Z^3 - Z^2 + 0.25 = 0$

Solution using Jury test:

① $F(1) = (1)^3 - (1)^2 + 0.25 = 0.25 > 0$ (Satisfied)

② $(-1)^3 F(-1) = (-1)^3((-1)^3 - (-1)^2 + 0.25) = 1.75 > 0$ (Satisfied)

③ $|a_0| = 0.25 < |a_n| = 1$ Satisfied

④ Construct Jury array

$$\begin{array}{|cccc} 0.25 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0.25 \\ \hline b_0 & b_1 & b_2 & \end{array}$$

$$b_0 = \begin{vmatrix} 0.25 & 1 \\ 1 & 0.25 \end{vmatrix} = -0.9375$$

$$b_2 = \begin{vmatrix} 0.25 & 0 \\ 1 & -1 \end{vmatrix} = -0.25$$

$|b_0| = 0.9375 > |b_2| = 0.25$ (Satisfied)

$\boxed{\text{The System is Stable}}$

Solution using Bilinear Transformation:

for c/c eqn $Z^3 - Z^2 + 0.25 = 0$, let $Z = \frac{1+r}{1-r}$

$$\left(\frac{1+r}{1-r}\right)^3 - \left(\frac{1+r}{1-r}\right)^2 + 0.25 = 0$$

$$(1+r)^3 - (1+r)^2(1-r) + 0.25(1-r)^3 = 0$$

$$1 + 3r + 3r^2 + r^3 - (1 + r - r^2 - r^3) + 0.25(1 - 3r + 3r^2 - r^3) = 0$$

$0.25 + 1.25r + 4.75r^2 + 1.75r^3$ → Construct Routh Array

$$\begin{array}{c|cc} s^3 & 1.75 & 1.25 \\ s^2 & 4.75 & 0.25 \\ s^1 & 1.16 & \\ s^0 & 0.25 & \end{array}$$

There is no Change in the Sign of 1st Column elements →

$\boxed{\text{The System is Stable}}$

ii) c/c eqn $Z^3 - 1.4Z^2 + 0.35Z + 0.05 = 0$

Solution using Jury test

① $F(1) = (1)^3 - 1.4(1)^2 + 0.35(1) + 0.05 = 0$ Not Satisfied (May Be Critical)

② $(-1)^3 F(-1) = (-1)^3\left((-1)^3 - 1.4(-1)^2 + 0.35(-1) + 0.05\right) = 2.7 > 0$ satisfied

③ $|a_0| = 0.05 < |a_n| = 1$ Satisfied

④ Construct Jury array

0.05	0.35	-1.4	1
1	-1.4	0.35	0.05
b_0	b_1	b_2	

$b_0 = \begin{vmatrix} 0.05 & 1 \\ 1 & 0.05 \end{vmatrix} = -0.9975$

$b_2 = \begin{vmatrix} 0.05 & 0.35 \\ 1 & -1.4 \end{vmatrix} = -0.42$

$|b_0| = 0.9975 > |b_2| = 0.42$ Satisfied

$\boxed{\text{The System is Critically Stable}}$

Solution using Bilinear transformation $Z = \frac{1+r}{1-r}$

c/c eqn: $\left(\frac{1+r}{1-r}\right)^3 - 1.4\left(\frac{1+r}{1-r}\right)^2 + 0.35\left(\frac{1+r}{1-r}\right) + 0.05 = 0$

$(1+r)^3 - 1.4(1+r)^2(1-r) + 0.35(1+r)(1-r)^2 + 0.05(1-r)^3 = 0$

$(1+3r+3r^2+r^3) - 1.4(1+r-r^2-r^3) + 0.35(1-r-r^2+r^3) + 0.05(1-3r+3r^2-r^3) = 0$

$2.7r^3 + 4.2r^2 + 1.1r = 0$

Construct Routh array

r^3	2.7	1.1
r^2	4.2	
r^1	1.1	
r^0	~~0~~ 1.1	

Aux eqn $P(r) = 1.1r$

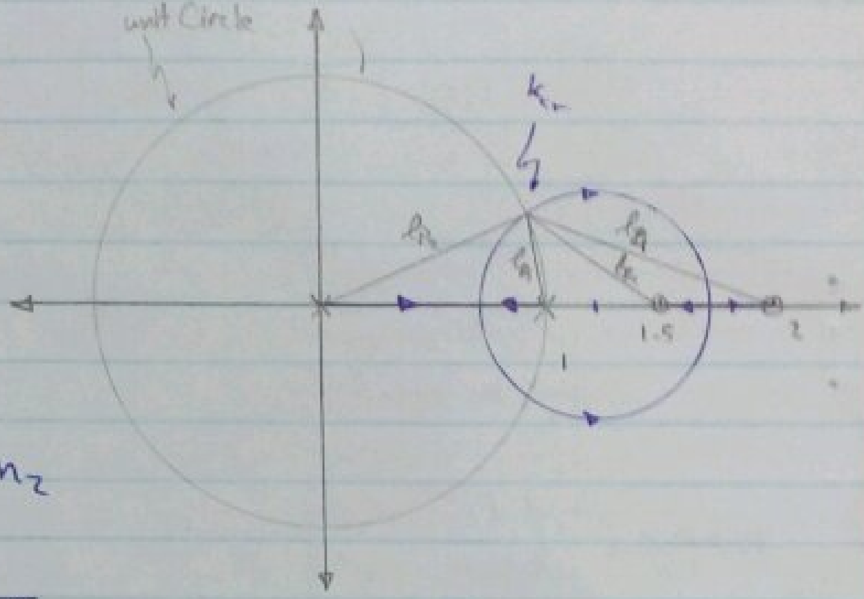
$\frac{dP(r)}{dr} = 1.1$

solution of $P(r) = 0 \rightarrow r = 0$

one pole on Jw axis

$\boxed{\text{The System is Critically Stable}}$

Problem no (2)



1- $n_p = 2 \longrightarrow Z = 0, 1$

$n_z = 2 \longrightarrow Z = 1.5, 2$

2- Real parts

To the left of odd number of poles and zeros

3- Asymptotes : Not found $n_p = n_z$

4- Breaking Points $k = \frac{-Z(Z-1)}{(Z-1.5)(Z-2)}$

Breaking Out (Max k)

Z	0.5	0.55	0.6	0.65	0.70	0.75
K	0.166	0.179	0.19	0.198	0.201	0.2

Breaking Out at $Z = 0.7$

Breaking in (Min k)

Z	1.51	1.55	1.6	1.65	1.7	1.75
K	157	37.8	24	20.4	19.8	21

Breaking in at $Z = 1.7$

5- No Departure angles

6- The root locus is a Circle with diameter $= 1.7 - 0.7 = 1 \longrightarrow r = 0.5$ and its Center at $1.7 - 0.5 = 1.2$

7- Range of k for stability

$$K_{cr} = \frac{l_{P_1} l_{P_2}}{l_{Z_1} l_{Z_2}} = \frac{1 * 0.4}{1.13 * 0.7} = 0.51$$

$$\boxed{0 < K < 0.51}$$

K_{cr} occurs at intersection between two Circles

$x^2 + y^2 = 1 \quad , \quad (x - 1.2)^2 + y^2 = 0.25$

$(x - 1.2)^2 + (1 - x^2) = 0.25 \longrightarrow x = 0.9125$

$\therefore y = 0.41 \longrightarrow K_{cr}$ at $Z = 0.9125 + j0.41$

$$K_{cr} = \frac{|Z||Z-1|}{|Z-1.5||Z-2|} = 0.504 \simeq 0.51$$